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Wormhole solutions in Einstein-Weyl gravity

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In this work we are going to study wormhole solutions in Einstein-Weyl gravity. Such solutions emerge when looking for a static spherically symmetric metric in the vacuum, in the more general context of classical quadratic gravity.

Classical quadratic gravity is the theory of gravitation that comes out when including quadratic terms in the curvature in the Einstein-Hilbert action of general relativity. The study of such theory is motivated by the presence of quadratic corrections in almost all the attempts to find a consistent description of quantum gravity. Indeed, it is well known that general relativity can be consistent as a quantum field theory only as a low-energy effective theory. We are not going to discuss the quantum aspects of the quadratic action: instead, we consider what happens to the classical description of the space-time when quadratic corrections are taken into account. In order to do that, we restrict to the simplest non-trivial case, that is a static spherically symmetric vacuum space-time. Given these restrictions in general relativity, we have the well known Schwarzschild solution, i.e. black hole solution. In classical quadratic gravity the Schwarzschild solution is still present, but we can also find many different classes of solutions: the aim of this thesis is to classify the various solutions families, as well as to characterize a specific family that covers a large part of the solutions space, i.e. the wormhole solutions.

We solved the geodesic equation in such solutions which shows the reason why we call them traversable wormholes: an observer that passes through the radius r_0 of the wormhole does not reach the region $r < r_0$, but ends up in a new copy of $r > r_0$. We reported all the solutions families found in the previous work and we added a new subfamily of the generic wormholes solutions that we have discovered in our analysis. In contrast to what happens in general relativity, the various solutions found in quadratic gravity are not always asymptotically flat.

When studying the different classes of solutions we are assisted by a Lichnerowicz-type theorem which removes the contributions of the R^2 term from the equations of motion under some assumptions, in particular when an horizon is present. When such contribution is absent, the quadratic theory reduces to Einstein-Weyl gravity. By numerically solving the equations of motion in the Einstein-Weyl theory, we have classified the various solutions families in a phase diagram of the theory, restricting to the asymptotically flat solutions. By using the shooting method for the boundary value problem between spatial infinity and the radius r_0, we found for the first time the geometric properties of the wormhole solutions, and in particular we characterized the behavior of these solutions in function of their position on the phase diagram. Then we used these results to explore both the interior $r < r_0$ of the wormholes and the new copy of $r > r_0$ that emerges in such solutions. We proposed a way to extend the solutions for the interior of the wormholes $r < r_0$, and we made an analysis of the metric in this region. However, a precise method to connect the interior and the exterior of such solutions is still lacking.

The last part is dedicated to explore the new region $r > r_0$, which is completely determined by the solutions around r_0. We have done a qualitative analysis of the behavior of the metric in this new copy of $r > r_0$, discovering that this new region has a finite volume for $r \to \infty$ for almost all the solutions, due to a strongly non-flat behavior of the metric.

Finally we concluded by showing what happens to an observer that falls into these solutions, finding that the surface $r \to \infty$ in the new region of the wormholes acts like a strongly attractive surface.